THERMOMETRY

Thermometry is the study of the principles used in the measurement of temperature.

Temperature

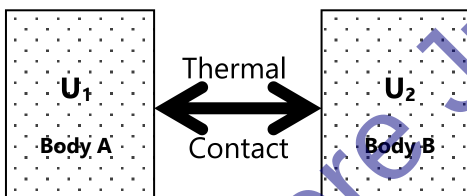
- ✓ This the quantity that is proportional to the internal energy of the body.
- ✓ Temperature is the measure of the degree of hotness or coldness of a body.

Heat

- ✓ Heat is the form of energy that is transferred from one object to another in thermal contact due to temperature difference between the objects.
- ✓ When heat is supplied to a body, the thermal (heat) energy of the molecules of the body increases and molecules vibrate with high speed hence their kinetic energy increases resulting into increase in temperature of the body.

Thermal equilibrium

Suppose that two bodies of internal energy (kinetic energy) U1 and U2 are in thermal contact.



- ✓ If the internal energy of body B is greater than that of body A (i.e. $U_2 \geq U_1$), them the molecules of body B are said to be more disordered than the molecules of body A. The molecules of B therefore have more kinetic energy and vibrate with greater amplitudes (velocity) than those of body A.
- ✓ The molecules of B thus collide with those of A and at every collision they transfer energy to molecules of A.
- ✓ This collusion continues until a point of thermal equilibrium, when the internal energy of both bodies are equal. I.e. the molecules of both bodies move with the same speed and vibrate with the same amplitude hence both bodies are at the same temperature.
- ✓ At this point the net rate of heat transfer is zero and the bodies are said to be in thermal equilibrium.

Temperature measurement and Scales

- ✓ Temperature is a quantity that is proportional to the internal energy of the body. $\left(T \propto K.E\right)$
- ✓ Temperature is measured using an instrument called a thermometer.
- ✓ Different thermometers use different substances/materials referred to as *thermometric substances*
- ✓ These different thermometric substances have different properties that are used in temperature measurement known as *thermometric properties*

Thermometric property

This is a physical property of a substance that varies linearly (*or uniformly*) and continuously with changes in temperature.

Examples of thermometric properties

Thermometric property	**Type of thermometer (application)**
Length or volume of a liquid in a capillary tube e.g. mercury	Liquid in glass thermometer e.g. clinical thermometer
Resistance of platinum wire	Resistance thermometer
The e.m.f of a thermocouple	Thermoelectric thermometers
Pressure of a fixed mass of a gas at constant volume	Constant volume gas thermometer
Volume of a gas at constant pressure	Constant pressure gas thermometer
Frequency or wavelength of radiation from hot bodies	Pyrometers

Fixed points

This is a single temperature at which a physical event is always expected to occur.

E.g. the boiling point of pure water at standard atmospheric pressure of 76cmHg is 100^0C

Types of fixed points

Lower fixed point: is the temperature at which pure ice can exist in equilibrium with pure water at standard atmospheric pressure.

Lower fixed points is also referred to as the *ice point* and occurs at 0^0C

Upper fixed point: is the temperature at which pure water can exist in equilibrium with its vapor at standard atmospheric pressure.

Upper fixed points is also referred to as the *steam point* and occurs at 100^0C

Triple point of water: is the temperature at which pure ice, pure water and pure water vapor can exist in equilibrium.
Triple point occurs at 273.16K

Types of temperature scales

The thermodynamic scale: This is a theoretical scale of temperature which is independent of the property of any particular substance. It is also referred to as the *absolute scale*

Steps in establishing the Thermodynamic scale

(i) Choose a thermometric property, say X
(ii) Obtain the value of the property at triple point of water X_{tr}.
(iii) Obtain the value of the property at unknown temperature T to be measured X_T
(iv) Express the temperature on the thermodynamic scale $T = \left(\frac{X_T}{X_{tr}} \times 273.16\right)K$

Example: The resistance of a platinum wire is found to be 2.8Ω at triple point of water and 2.2Ω at unknown temperature of a body. Find the temperature of the body. *(214.6K)*

Absolute Zero

- ✓ This is the temperature when the molecules of the gas have zero kinetic energy and is assumed to occupy zero volume.
- ✓ Absolute zero temperature of an ideal gas corresponds to its zero volume or zero pressure it exerts on the walls of the container in which it is trapped. This value approximates to the triple point of pure water i.e. -273^0C or 0K.

Molecular explanation for existence of absolute zero temperature

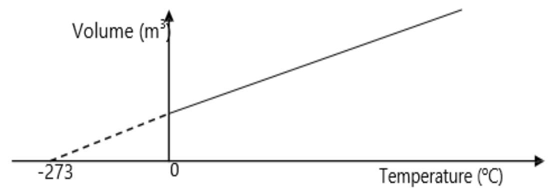
- ✓ When a gas is cooled, its molecules loose kinetic energy continuously since it depends directly on temperature.
- ✓ As molecules loose kinetic energy they move closer to each other until when they cease to have kinetic energy and their motion stops.
- ✓ At this point the gas is said to occupy a negligible volume and its temperature at this point is called the absolute zero temperature and the pressure the gas exerts on the walls of the container occupied is negligible.

Estimating absolute zero temperature

A given volume of a fixed mass of an ideal gas is trapped and cooled. During cooling, the gas molecule slow down since their average kinetic energy entirely depends on their thermal dynamic temperatures.

Molecules move closer until when their mean kinetic energy is zero i.e. molecules cannot move any more. When a plot of gas volume "V" is plotted against its Celsius temperature, the following plot is obtained.

Absolute zero temperature is determined by extrapolating the graph until when it touches the temperature axis and is found to be **-273 C or 0K.**



The Celsius scale

Celsius scale measures temperature in degrees Celsius (^0C) and is denoted by $\theta = T - 273$ where θ is the temperature in (^0C) and T the temperature in Kelvin.

Note: The SI unit of temperature is the Kelvin (K) and is defined as $\left(\frac{1}{273.16}\right)$ of the temperature of the triple point of water as measured by the thermodynamic scale.

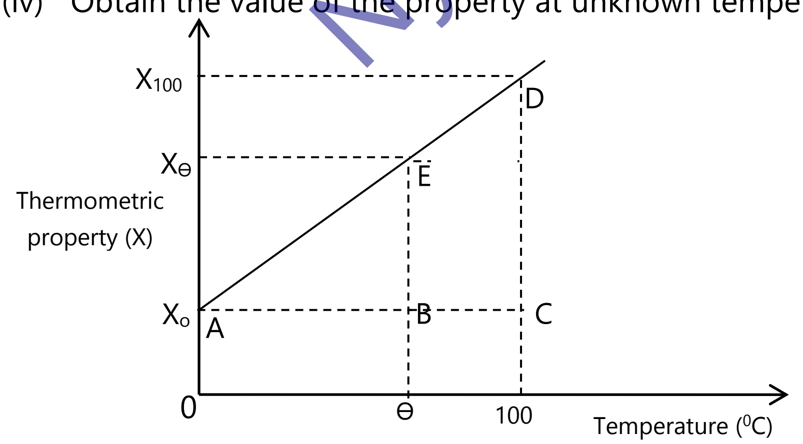
Steps in establishing the Celsius scale

(i) Choose a thermometric property, say X

(ii) Obtain the value of the property at ice point of water X_o

(iii) Obtain the value of the property at steam point of water X_{100}

(iv) Obtain the value of the property at unknown temperature θ to be measured X_θ



By comparison of slopes

Slope of Δ ABE = slope of Δ ACD

$$\frac{BE}{AB} = \frac{CD}{AC}$$

$$\frac{X_\theta - X_0}{\theta - 0} = \frac{X_{100} - X_0}{100 - 0}$$

$$\Rightarrow \theta = \left(\frac{X_\theta - X_0}{X_{100} - X_0}\right)100^0C$$

(v) On the Celsius scale, θ is calculated from $\theta = \left(\frac{X_\theta - X_0}{X_{100} - X_0}\right)\times 100^0C$

Examples

1. The resistance of a platinum wire is found to be 2.80Ω and 3.40Ω at the ice point and steam point respectively. When the thermometer is in contact with a body, its resistance is found to be 3.04Ω.

Solution $R_0 = 2.80\Omega,\ R_{100} = 3.40\Omega,\ R_\theta = 3.04\Omega \quad \Rightarrow \theta = \left(\frac{R_\theta - R_0}{R_{100} - R_0}\right) \times 100^0C \therefore \theta = 40^0C$

2. The length of mercury in a mercury in glass thermometer is found to be 15cm at ice point and 75cm at steam point. When the bulb of the thermometer is immersed in a cup of tea, the length is found to be 54cm. find the temperature of the tea. (*65°C*)

3. The resistance of platinum thermometer is 2.04Ω at ice point and 3.02Ω at the steam point. Find the resistance of the platinum wire when immersed in molten salt at 735°C. *(9.243Ω)*

The liquid in glass thermometer

- ✓ This is the most common type of thermometer
- ✓ Thermal expansion of the liquid in the thermometer as a result of temperature increase causes the liquid thread in the stem to increase in length.

Mercury is used because:

(i) It is opaque and therefore easily seen

(ii) Mercury is a good conductor of heat and therefore responds to temperature changes in the environment.

(iii) It expands almost uniformly with increase in temperature.

(iv) It does not wet/stick to glass

(v) Its temperature range is sufficient for normal use i.e. $\left(-39^0C \textit{ to } 357^0C\right)$

The liquid in glass thermometers however have relatively large heat capacities and this limits their use in two ways;

(i) They cannot be used to measure rapidly changing temperatures

(ii) They affect the temperature of the body whose temperature is to be measured.

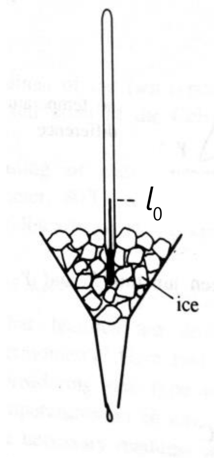
The accuracy of a liquid in glass thermometer during use as a thermometer is affected by the following

(i) Non uniformity of the bore

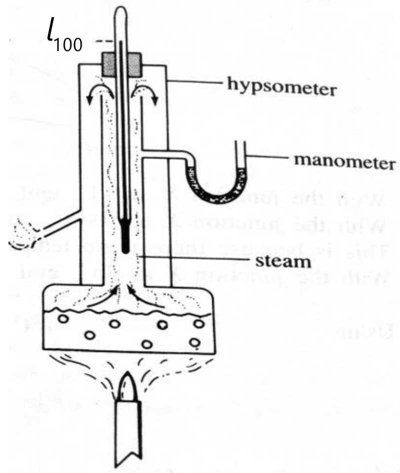
(ii) The glass expands and contracts before heat is conducted to the liquid

(iii) Parallax errors when reading the scale.

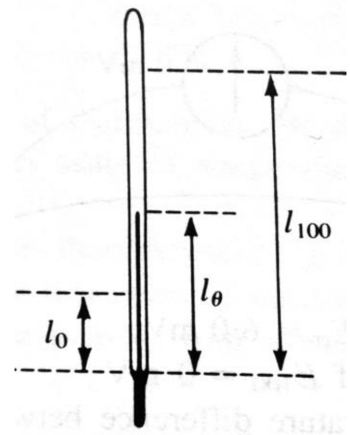
Calibration of a liquid in glass thermometer



(a)



(b)



(c)

To calibrate a thermometer, the following procedures are taken

(i) The lower fixed point

The bulb of the thermometer is placed in pure melting ice in a funnel *(fig a)*

When the mercury level in the thermometer is stable, its level is marked on the stem

Measure this length l_0 from the bulb

(ii) The Upper fixed point

The bulb of the thermometer is placed in pure steam from pure boiling water at standard atmospheric pressure *(fig b)*

When the mercury level in the thermometer is stable, its level is marked on the stem

Measure this length l_{100} from the bulb

(iii) Fundamental interval

The distance between the two levels marked on the stem of the thermometer is then divided into 100 equal divisions.

To measure temperature θ^0C when the length of the mercury thread is l_θ is

given by $$\theta = \left(\frac{l_\theta - l_0}{l_{100} - l_0}\right) \times 100\,^0C$$

Advantages of liquid in glass thermometer

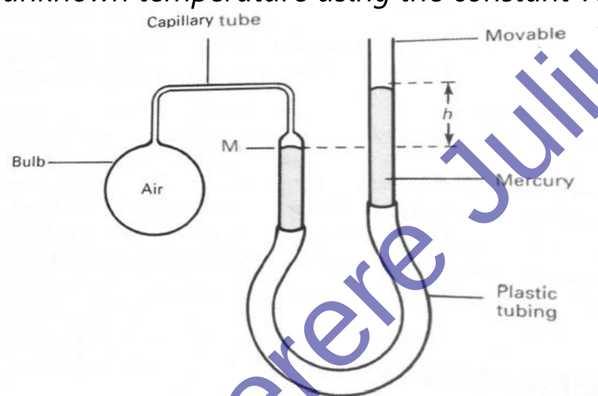
(i) Easily and quickly read *(since temperature is read directly)*
(ii) Very portable
(iii) Cheap to buy
(iv) Simple and suitable for normal use

Disadvantages of liquid in glass thermometer

(i) Less accurate
(ii) Small/limited range of temperature measurements
(iii) Can easily break
(iv) Cannot measure rapidly changing temperature because of the high heat capacity

The constant volume gas thermometer

Measurement of unknown temperature using the constant volume gas thermometer



- ✓ The bulb is placed in a medium whose temperature is to be measured and left for some time to acquire the temperature of the medium
- ✓ The gas in the bulb expands and pushes the mercury up in the flexible limb
- ✓ The flexible limb is adjusted to bring back the mercury level to the constant volume mark M.
- ✓ The difference in the mercury levels of h is measured
- ✓ The pressure of the gas P_θ is calculated from $P_\theta = P_A + h\rho g$ where P_A is the atmospheric pressure.
- ✓ The bulb is then immersed in an ice – water mixture and the pressure P_0 calculated.
- ✓ The bulb is also immersed in steam from boiling water at standard atmospheric pressure and the corresponding pressure P_{100} calculated.
- ✓ The unknown θ temperature is calculated from; $\theta = \left(\frac{P_\theta - P_0}{P_{100} - P_0}\right) \times 100^0C$

Sources of errors

(i) The air in the capillary tube *(called dead space)* is not at the same temperature as that in the bulb.

(ii) Air is not an ideal gas

(iii) The gas bulb also expands or contracts before the gas expands or contracts

(iv) Non – uniformity of the bore of the capillary tube

Corrections

To improve on the accuracy of the constant volume gas thermometer, the following should be done;

(i) The dead space should be narrowed

(ii) The gas bulb must be made of hard glass

(iii) The gas bulb should be made from highly conducting material

Note: the constant volume gas thermometer is used to calibrate other thermometers because it's very sensitive and accurate.

Advantages of constant volume gas thermometer

(i) It is very sensitive

(ii) It is very accurate

(iii) It can easily be constructed

(iv) It has a wide range of temperature measurements i.e. $\left(-270^0C \text{ to } 1{,}500^0C\right)$

(v) It is used for calibrating other thermometers

Disadvantages of a constant volume gas thermometer

(i) It is very difficult to use and time consuming

(ii) It does not give direct reading

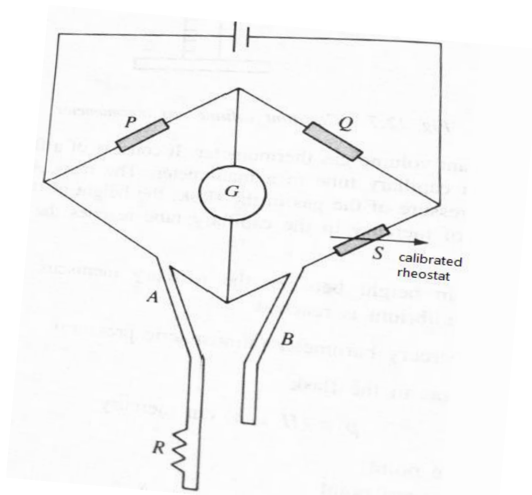
(iii) It is cumbersome

(iv) It cannot measure rapidly changing temperature

(v) It cannot measure temperature at a point because of the wide base of the bulb

Platinum resistance thermometer

- ✓ The thermometric property used is the resistance of the coil of platinum wire.
- ✓ Platinum wire is used in resistance thermometer because it has a high melting point and high temperature coefficient of resistance



- ✓ The resistance R of the platinum coil is measured using a Wheatstone bridge circuit in which P and Q are standard resistors.
- ✓ When the leads A and B are immersed in a medium whose temperature θ is to be measured, the rheostat is varied until the galvanometer G gives no deflection. I.e. at balance state.
- ✓ The resistance R_θ at unknown temperature θ is obtained from;

$$\frac{R_\theta}{S} = \frac{P}{Q} \qquad \therefore R_\theta = \frac{P}{Q} \times S$$

- ✓ The leads are then immersed in an ice – water mixture and the resistance R_0 calculated.
- ✓ The leads are also immersed in steam from pure boiling water at standard atmospheric pressure and the resistance R_{100} calculated.
- ✓ 'the unknown θ temperature is calculated from; $\theta = \left(\frac{R_\theta - R_0}{R_{100} - R_0}\right) \times 100^0C$

Note:

(i) The dummy leads are used to compensate for any change in the resistance in the leads to the platinum coil when the temperature changes.

(ii) The dummy leads are made of the resistance as the copper leads

Advantages of platinum resistance thermometer

(i) It is accurate since the resistance of platinum wire varies linearly with temperature.

(ii) It gives wide range of temperature measurements i.e. $\left(\text{-}180^0C \ to \ 1{,}500^0C\right)$

(iii) It is most suitable for measuring constant temperatures

Disadvantages of platinum resistance thermometer

(i) It cannot measure temperature at a point.
(ii) It cannot measure rapidly changing temperatures
(iii) It is slow to respond since platinum has a high specific heat capacity

Example: A resistance thermometer has resistance of 9.97Ω at ice point and 14.04Ω at steam point. Find the temperature on the Celsius scale of this thermometer when its resistance is 4.60Ω. *(-131.94^{0}C)*

Note: *When resistance thermometer is calibrated using a constant volume gas thermometer, the resistance R_θ of platinum is found to vary with Celsius temperature θ according to the equation $R_\theta = R_0\left(1+\alpha\theta+\beta\theta^2\right)$ where R_0 the resistance of platinum at 0^0C is while α and β are constants.*

Example:

The resistance R_θ of a platinum resistance thermometer at a Celsius temperature θ as measured by a constant volume gas thermometer is given by;

$$R_\theta = 50.00 + 0.17\theta + 3.00\times10^{-4}\theta^2$$

Calculate the temperature as measured on the scale of the resistance thermometer which corresponds to a temperature of 60^{0}C on the gas thermometer

Approach

$R_\theta = 50.00 + 0.17\theta + 3.00\times10^{-4}\theta^2$

$R_{100} = 50.00 + 0.17\times100 + 3.00\times10^{-4}\times100^2$

$R_{100} = 70.00\Omega$

$R_0 = 50.00 + 0.17\times0 + 3.00\times10^{-4}\times0^2$

$R_0 = 50.00\Omega$

$R_{60} = 50.00 + 0.17\times60 + 3.00\times10^{-4}\times60^2$

$R_0 = 61.28\Omega$

$$\theta = \left(\frac{R_\theta - R_0}{R_{100} - R_0}\right)\times100^0C$$

$$\theta = \left(\frac{61.28-50.00}{70.00-50.00}\right)\times100^0C$$

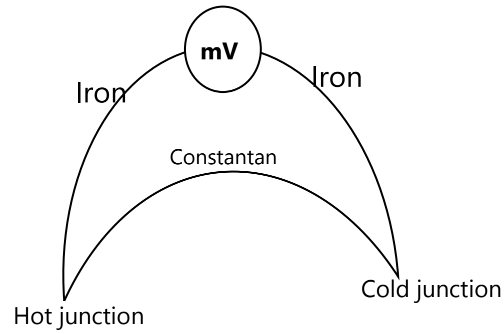
$$\theta = 56.4^0C$$

2. The value of property X of a given substance is given by

$$X_\theta = X_0 - 0.5t + 2.0\times10^{-4}t^2$$

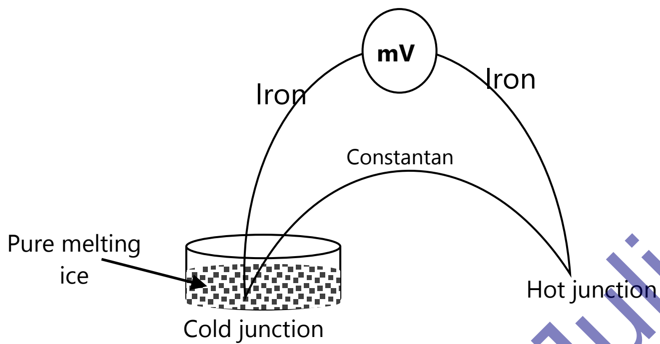
Where *t,* is the temperature in ^{0}C measured by the gas thermometer scale. What would be the Celsius scale temperature in ^{0}C defined by the thermometer scale? *(51.04^{0}C)*

Thermoelectric thermometer



- ✓ A thermocouple consists of two wires of different metals joined at two junctions. If the two junctions are kept at different temperatures, a small e.m.f is set up.
- ✓ The magnitude of this e.m.f increases with the increase in the difference between the two junctions.
- ✓ The e.m.f generated can be measured using a *millivoltmeter* or a *potentiometer*

Measurement of temperature using thermocouple thermometer



- ✓ The cold junction is kept at a fixed reference point at pure melting ice.
- ✓ The hot junction is placed in the body whose temperature θ is to me measured and the resulting e.m.f E_θ is recorded.
- ✓ The junction is also inserted in an ice – water mixture and the e.m.f E_0 recorded.
- ✓ The junction is also inserted in steam from pure boiling water at standard atmospheric pressure and the e.m.f E_{100} recorded.
- ✓ The unknown temperature is calculated from; $\theta = \left(\frac{E_\theta - E_0}{E_{100} - E_0}\right) \times 100^0C$

Advantages of a thermoelectric thermometer

(i) It can measure rapidly changing temperature. This is because the wires used are thin and have low value of specific heat capacity.

(ii) Thermocouples measure temperatures at a point such welded joints.

(iii) It is sensitive

(iv) It has wide range of temperature measurements i.e. $\left(-250^0C \text{ to } 1{,}500^0C\right)$

(v) They are cheap and easy to use

Disadvantages of thermocouple thermometers

(i) Thermocouples are less accurate

(ii) E.m.f of a thermocouple does not vary linearly with temperature

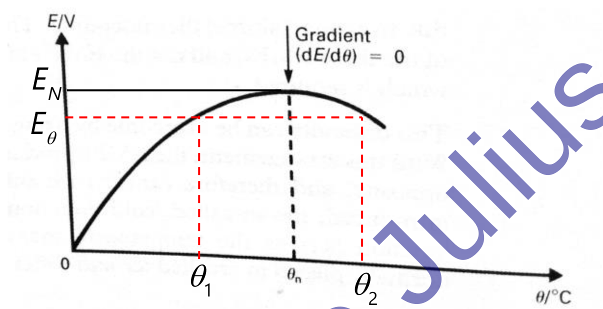
Variation of e.m.f with temperature in thermocouples

The e.m.f of a thermocouple is found to approximate the equation $E = \alpha\theta + \beta\theta^2$

Where α and β are constants dependent on the pair of metals used.

The above equation of parabolic and therefore there exists a temperature for which $\frac{dE}{d\theta} = 0$.

This temperature θ_n is called *neutral temperature*.



Values of temperature measured by the thermocouple near the neutral temperature are not accurate because

(i) the variation of e.m.f with temperature is small and the thermocouple is insensitive

(ii) the thermocouple can give similar readings of e.m.f corresponding to two different temperatures *(see the red dotted lines)*

Neutral point of thermocouple is therefore the maximum temperature of thermo couple just below which its e.m.f's vary fairly linearly.

Thus thermo couples are only used as thermometers at temperatures below its neutral temperature.

Pyrometers

Pyrometers use the quality of the wavelength of the radiation emitted by hot bodies as their thermometric property.

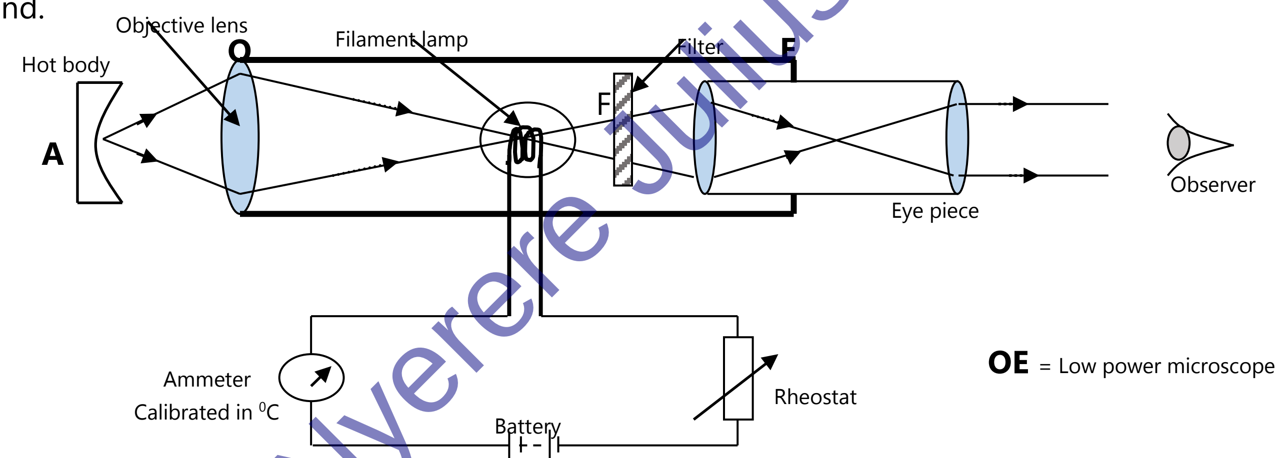
These emitted radiations can either fall on the visible or the invisible part of the electromagnetic spectrum, and for this reason, pyrometers are of two types, namely;

(i) Optical pyrometer: this responds to radiations only in the visible part of the E.M spectrum

(ii) Total Optical Pyrometer: this responds to radiations both in the visible and invisible part of the E.M spectrum

Optical Pyrometer *(Disappearing Filament Pyrometer)*

In this type of pyrometer, the tungsten filament of an electric bulb is used as a radiator. The intensity of radiation of filament is compared with the intensity of the radiation of the hot surface. When both intensity match, the filament disappears against the back ground.



__Determination of the temperature of a hot body such as furnace, hot fire source__

- ✓ The hot body is focused using the objective lens such that its image lies in the same plane with that of the filament.
- ✓ If the filament appears brighter on a dark background, then the filament is hotter than the hot body.
- ✓ If the filament appears darker on a brighter background, then the hot body is hotter than the filament.
- ✓ In either cases, the rheostat is adjusted until the appearance of the filament disappears in the background.
- ✓ At this point, both the filament and the hot body are at the same temperature.
- ✓ The temperature is then read off from the ammeter which is calibrated in degrees Celsius directly.

Total Optical Pyrometer

The total radiation pyrometer receives all the radiation from a particular are of hot body. The term total radiation includes both the visible and invisible radiations. It consists of radiation receiving element and a measuring device.

>>> Diagram

>>> Mode of operation

Sample questions

1. The resistance of a substance at a temperature t^0C which is measured by mercury thermometer is $R_t = \dfrac{R_o}{1+\alpha t^2}$

Where R_0 is the resistance at 0^0C and $\alpha = 5.0\times10^{-5}\,{}^0C^{-2}$. What is the reading of the resistance thermometer when the reading of the mercury thermometer is 100^0C and 80^0C? Comment on your answers.

Solution $R_{100} = 0.6667R_0$ $R_{80} = 0.7576R_0$

When the mercury thermometer reads 100^0C, the temperature as recorded by the resistance thermometer is $\theta = \left(\dfrac{R_{100}-R_0}{R_{100}-R_0}\right)\times100\,{}^0C \Rightarrow \theta = 100\,{}^0C$

When the mercury thermometer reads 80^0C, the temperature as recorded by the resistance thermometer is $\theta = \left(\dfrac{R_{80}-R_0}{R_{100}-R_0}\right)\times100\,{}^0C \Rightarrow \theta = 72.7\,{}^0C$

Comment: The readings of the two types of thermometer are the same for 100^0C because 100^0C is the upper fixed point of the Celsius scale of temperature. All thermometers agree at the fixed points.

The reading of the resistance thermometer is 72.7^0C, differs from that of mercury thermometer, 80^0C, because the resistance of the substance and the expansion of mercury do not vary in the same manner with increase in temperature.

2. A thermometer is constructed with a liquid which expands according the relation $V_t = V_0(1+\alpha t+\beta t^2)$ Where V_0 and V_t are the volumes at 0^0c and at t^0c on the scale of a gas thermometer and given that $\alpha = 1000\beta$, what will the thermometer read when the gas thermometer reads 50^0c?

3. (a) What is meant by the following terms as applied to thermometry?

(i) Kelvin (ii) Triple point

(b) The electrical resistance R_T of a certain thermometer varies with temperature **T**, in kelvin according to the equation $R_T = R_o\left[1+5.0\times10^{-3}(T-T_0)\right]$ Where R_0 is the resistance at T_0. Given that the resistance at the triple point of water is 101.6Ω and 165.5Ω at 600.5K, what is the temperature when the resistance is 132.5Ω?

(c) (i) What is meant by a thermometric property?

(ii) List four thermometric properties which are used in thermometry.

(ii) The resistance of a thermistor over a limited range of temperature is given by the equation; $R = \frac{c}{T-203}$

Where, c, is a constant and T is the absolute temperature.

Calculate the temperature on the centigrade scale of the thermistor at an absolute temperature, T = 310 K.